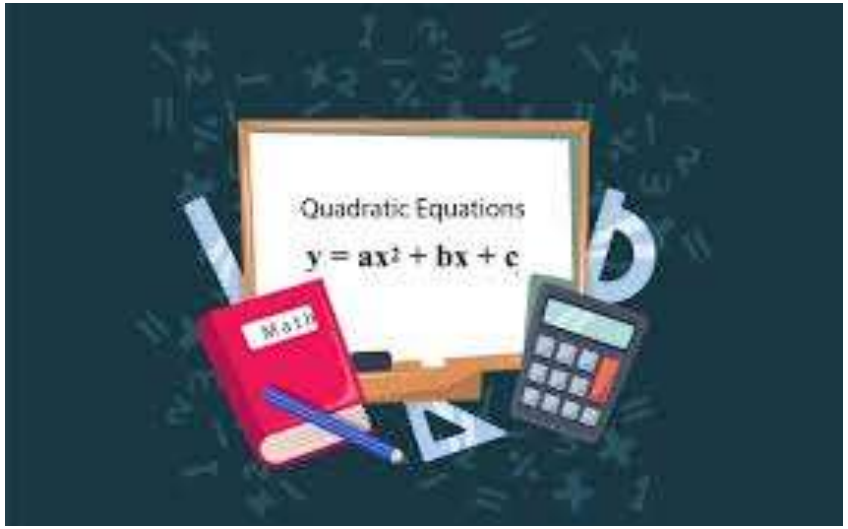
Quadratic Equation



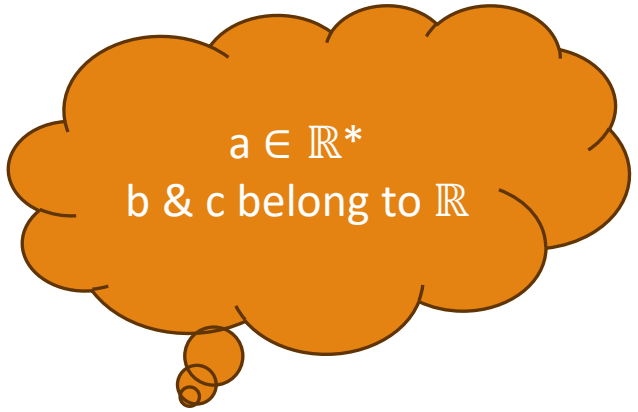
DEFINITION

METHOD OF COMPLETING SQUARES

What is a quadratic equation ?

A quadratic equation in x is a second-degree equation that can be written in the form :

$$ax^2 + bx + c = 0$$



Quadratic Equation : Examples

$2x^2 - 3x + 1 = 0$

$-5x^2 + 2x - 9 = 0$

$8x^2 - 3x = 0$

$2x^2 + 1 = 0$

How to solve quadratic equation ?

Methods to solve quadratic equations

- Factoring
- Methods of completing squares
- Quadratic Formula
- Graphing

Method of completing squares

Recall

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a-b)(a+b)$$

Method of completing squares

$x^2 + 6x + 5 = 0$

Divide the coefficient of x by two and square it

$\frac{6}{2} = 3 \rightarrow 3^2 = 9$

$x^2 + 6x + 9 - 9 + 5 = 0$

Add and subtract this square

Method of completing squares

$x^2 + 6x + 9 - 9 + 5 = 0$

The first three terms form a perfect square

$(x+3)^2 - 9 + 5 = 0$

$(x+3)^2 - 4 = 0$

Now apply the third remarkable identity

$(x+3)^2 - 2^2 = 0$

$(x + 3 - 2)(x + 3 + 2) = 0$

Method of completing squares

$$(x + 3 - 2)(x + 3 + 2) = 0$$

$$(x + 1)(x + 5) = 0$$

$$(x + 1) = 0 \qquad (x + 5) = 0$$

$$x = -1 \qquad x = -5$$

Notes

$(x + 3)^2 + 4 = 0$

$(x + 3)^2 = -4$

Impossible

Notes

$2x^2 - 3x + 1 = 0$

Divide both sides by a

$x^2 - \frac{3}{2}x + \frac{1}{2} = 0$

Do same steps before!

Time for Practice

Solve the following quadratic equations using the method of completing squares :

a) $x^2 - 8x + 12 = 0$

b) $x^2 + 5x + 7 = 0$

c) $3x^2 + 6x - 9 = 0$

Time to check your work

Equations	Answer
a) $x^2 - 8x + 12 = 0$	$x = 6 \quad or \quad x = 2$
b) $x^2 + 5x + 7 = 0$	No solution
c) $3x^2 + 6x - 9 = 0$	$x = -3 \quad or \; x = 1$